

Module 8: Economic Decision: Time value of money and economic decisions

Learning Goal for Module 8

“Time is money,” or so the saying goes. In this Module, we invert this to say “money is time.” Our valuation of money depends on when we receive it. Because life is uncertain, we usually prefer income now, unless we receive additional payment in the form of interest. The time value of money forms the foundation for most economic decisions.

By the end of this Module, you will understand:

- The concept of interest from multiple perspectives
- Ways to value a stream of income from an investment
- Ways to value different income streams
- The core idea behind the cost-benefit model

1. Introduction

Economic theory has treated the concept of interest from many perspectives. Marxian economics views interest as the outcome of capitalist investment (or accumulation), rising and falling with the business cycle. The Austrian economists viewed interest payments as a reward for deferring income. Other perspectives view interest as a risk premium for accepting the uncertainty of repayment. The treatment in this Module adopts the last two perspectives, where interest is reward/compensation for delaying gratification and accepting risk. Now the question becomes “How are time and interest connected?” This is the core theme of this Module.

Little occurs instantaneously, although events at the sub-atomic level may occur very quickly. In economics, investment decisions require time to assess options, mobilize the resources, and monitor the results.

- *Example:* Imagine you are a graphic designer, and your friend is an internet wiz. You decide to create a business to help traditional brick and mortar businesses (pizza joints, children’s clothing stores, etc.) move from serving customers in person to creating an online shopping capacity. You develop a business plan that requires purchasing several computers plus a server, assuming the costs of high-speed internet, plus your time in designing the support systems (user interface, order taking, payment fulfillment, etc.). You also may need a loan for some equipment, or you might lease computers. Your first months in business are a stream of costs without any revenue. At some point, you need customers, and then revenues start to build and hopefully exceed costs. How long do you wait before you decide the venture has succeeded or failed? Is there a way to structure information to help you decide whether even to start this business?

Module 8: Economic Decision: Time value of money and economic decisions

- *Example:* A pharmaceutical company needs to decide whether to invest in a novel approach to creating a COVID antibody test that you wear as a patch, shows results within minutes of the infection starting, and is reliable (low rates of false positive/negative results).

This Module considers two themes:

- *Time preference (interest).* Interest payments build on the idea that getting something now is better than getting it in a year. Of course, it is much more complicated than that, but the simple formula that relates the value of a dollar in your pocket now to the value (now) of getting that dollar in a year forms the foundation of all financial concepts from mortgages to comparing the choice between investing in that new internet B2B idea.
- *Economic valuation.* Economic decision frameworks all start with comparing the value now (present value) coming from multiple income streams stretching into the future. The investment portfolio choice requires valuating many different options. In venture capital, how does one compare investment A, with a large initial cost producing a steady annual revenue, with a second investment that has the same revenues, but continuous costs? In health, how do we decide between creating new hospital spaces versus increasing training? Should government invest in charging stations?

1.1. Time preference and investing

Time preference refers to the valuation of today's consumption relative to tomorrow. Rationing is a common application, such as when castaways in a lifeboat ration scarce water and food to prolong life.

- Students who divert resources from today's consumption to pay for tuition and living costs at university expect/hope that the investment will increase their income and make the sacrifice/investment worthwhile.
- Someone with a *positive* time preference prefers today's consumption over tomorrow's; those who are neutral are indifferent between consuming today or tomorrow, while those with a *negative* time preference are willing to forego consumption today under the expectation that such diversion will increase the capacity to consume tomorrow.

Module 8: Economic Decision: Time value of money and economic decisions

- Since a certain amount of risk exists as to whether any of us will be alive tomorrow – a risk that increases with age and other factors – this may always create a slightly positive time preference in all of us. Some authors refer to those with high positive time preference (no clear idea what constitutes “high”) as having time “myopia” or “faulty telescopic faculty.” This is pejorative. Those whose experience has not offered rewards from waiting are likely to value current consumption more highly than saving/investing. At the extreme, those who do not expect to live long due to terminal illness or war have strong positive time preferences.

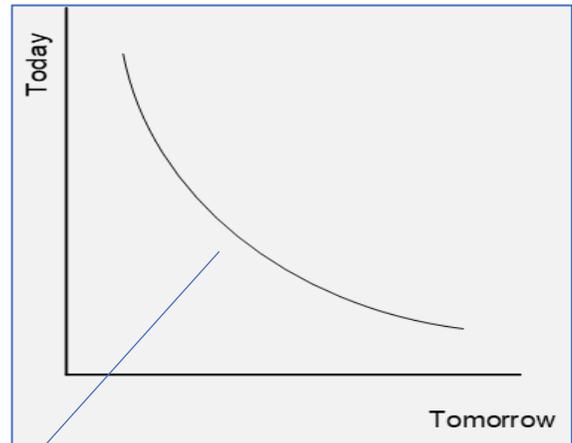


Figure 1: Time preference

But why does it curve rather than being a straight line?

From a consumer’s perspective, if you take resources from today’s consumption, then the remaining consumption resources are more valuable. Every additional dollar shifted from today becomes more valuable than the previous dollar invested. From the investor perspective, one invests in the higher return projects first. Therefore, every additional dollar shifted from today to tomorrow is less productive than the previous dollar.

The famous “[marshmallow experiment](#)” purportedly shows that negative time preference (willing to wait to get a higher reward) occurs at an early age. Children with high negative time preference are believed to be more successful because they are willing to defer immediate pleasure to invest in education, work on a career, etc. [Recent research](#) suggests that the situation is more complex and that this famous example is simplistic.

1.2. Why do interest rates exist?

Why do most people accept that, when they lend money, they should receive interest, and when they borrow, they should pay interest? In fact, certain religious principles condemn interest. The Christian laws of usury in the middle ages forbade any lending/borrowing that involved charging interest. Today, Sharia law forbids charging or accepting interest. Islamic bankers get around this through a complex series of buy/sell agreements between investor and bank, where interest becomes submerged in the difference between the buying and selling prices.

In western finance, “usury” may refer to charging “exorbitant” interest. Of course, whether a specific interest is exorbitant is a matter of opinion and context. With current interest rates

Module 8: Economic Decision: Time value of money and economic decisions

climbing, exorbitant rates might be in the order of 10%. Twenty years ago, when annual inflation was running at 15%, some may say that interest would need to exceed 20% to be usurious. Payday lenders often receive criticism due to high interest rates, and credit cards routinely charge over 20% on unpaid balances. They get around usury laws by registering corporations in states with lax regulation of maximum interest rates, a reason why Delaware has so many financial firms registered in that state.

1.3. Effective vs. nominal rates

In simple problems, interest is usually quoted in annual rates. Often, specific situations call for part-year investments, and we need to distinguish between

- nominal annual rate, and
- annual equivalent rate (AER) or “effective rate.”

The formula is

$$\text{AER} = \left(1 + \frac{r}{n}\right)^n - 1$$

- **Example:** You are offered two investments: A, which has a nominal interest of 10% compounded monthly, and B, which has a nominal interest of 10% compounded semi-annually. Which has the higher return?
Answer: Use the formula above to obtain A = 10.47%, and B=10.36%. The Excel function is =EFFECT(nominal rate, npery), where the nominal rate is the annual rate and npery is the number of compounding periods per year.
- **Example:** What is the interest payable on \$3,000 invested for six months at a nominal annual rate of 10%, and then for two years at a nominal annual rate of 7%?
Answer: Step 1 – Calculate the return from the first period as \$150. Step 2 – Invest the principal plus interest for two years, so \$3,150 $(1.07)^2 = \$3,407.04$.
- **Example:** If the nominal annual rate is 6%, then the monthly rate is .5%, and an investment of \$1,000 will earn \$1,061.68 in 12 months compared to the \$1,060 that would be received with an annual rate of 6%.

See

[Effective_Nominal.xlsx](#)

1.4. Simple interest

Future value: When the principal, P, earns interest for its entire life (basic savings account), then in Year t, $F = P*(1+i)^t$.

- **Example:** If interest is 5%, then in one year, \$100 will earn $\$100*(1+.05) = \105 .

$$F = P(1 + i) = 1.05 = 100(1.05)$$

Module 8: Economic Decision: Time value of money and economic decisions

- **Example:** If we reinvest the \$105, by the end of the second year, the investment will earn \$105 + \$105*(1+.05) = 110.25 = \$102*(1.05)²– this is termed **compounding**.

$$F = P(1+i)(1+i) = P(1+i)^2 = 110.25 = 100(1.05)(1.05)$$

- **Example:** When COVID cases double every two days, we see exponential growth, which is the same as compounding.

Present value refers to the idea that future cash received or paid in the future is worth less than cash received or paid now. The present value of an amount F received in t years when the interest i is given by the formula

$$F = P(1+i)^n$$

- **Example:** A debt with a face value (value printed on the bond/loan) of \$1,200 and annual interest of 5% due in five months is currently worth $P = \$1200 / (1 + 5\% * 5/12) = \1175.51 . Note that we are combining the idea of effective interest rate and present value in a single example. Interest quoted annually as 5% is *effectively* $5\% * (5/12) = 2.08\%$.

$$P = F / (1+i)^n$$

This single formula links present and future values and lies at the heart of financial economics and investment decisions. An investment decision is much more than buying stocks or bonds; fundamentally it means any decision where one decides on how to allocate scarce resources, including time, to a range of competing alternatives. Do I borrow to purchase equipment for my new web business or to build a mobile kitchen to deliver contactless food? Or maybe I should not borrow anything to set up a business and practice my gaming skills in my parents' basement!

See the following:

[Present Value Example.xlsx](#)

[Present Value Models.xlsx](#)

[Present Value Tables.xlsx\]](#)

This Module has many examples with notes in the Excel worksheets. Make sure you go through these.

Module 8: Economic Decision: Time value of money and economic decisions**1.5. Interest rates and asset prices**

One of the first things any investor learns is that interest rates and bond prices are inversely related. Imagine a bond that has a face value of \$100 with an annual interest rate of 5% earning \$5 annually. If interest rates are steady, the investor has no cause for concern.

Imagine, however, that interest rates increase to 6% for similar types of bonds. The investor cannot simply cash in the bond and transfer the money to another bond with 6% interest rate. They must **sell** that bond, but who would want to buy it for \$100 with a face interest of 5% when one could buy a current bond for \$100 carrying a 6% interest? The “old” bond price must decline so that its interest (5%) will equal \$6, which means the “old” bond must sell for \$83.33. (To check: $\$5/\$83.33=6\%$ - approximately).

One-year bonds. A one-year bond that makes a single payment of \$X in one year (principal plus interest) will have a price, call it P_{xi} , that depends on the market interest rate, i , according to

$$P_{xi} = X / (1+i)$$

Perpetuities. A bond that pays an annual amount forever, without paying back the original principal, is a “perpetuity”. *Example:* A perpetuity of \$50 per year, that you spent \$1,000 to obtain, has a 5% interest. If someone pays you \$1,050 to obtain that \$50/year, then the implicit interest is $\$50/\1050 or 4.76%. The relationship between interest rates and the price of a perpetuity is

$$P_{xp} = X / i$$

The present value formula connects the value of an income-producing asset, time, and interest. By widening the concept of income to include utility, the valuable assets that produce a stream of non-monetary benefits also change with interest and time. The “trick” is to determine how to translate things, such as the pleasure of owning one’s own home, or rate art into monetary terms. Usually, this involves a measure of “willingness to pay” money, which seems a little circular...but that is beyond the scope of this course.

The higher the interest rate, the greater the discount on a future return.

\$100 received in one year when interest is 5% is presently worth

$$PV = \$100/(1+.05) = \$95.24$$

\$100 received in one year when interest is 15% is presently worth

$$PV = \$100/(1+.15) = \$86.96$$

This is intuitive – if interest rates are high, we are less patient with delayed payment since we could use the cash right now and invest in savings with a high return. The further into the future the payment, the more we need compensation to accept the delay and the risk.

This applies to fully informed and rational investors that do not face unusual circumstances (such as being informed they have a terminal disease and have six months to live).

Module 8: Economic Decision: Time value of money and economic decisions

2. Comparing income streams

To return to the above example, how does one compare two investments, one with a high single upfront investment producing a steady stream of revenue and one that requires a smaller continuous cost to produce the same revenue? Two common methods are

- payback period, and
- discounted net cash flow (present value).

Payback period

A quick method for comparing the return on projects.

The faster you get your investment back, the higher the value of that investment.

$$Payback = (Initial\ Investment) / (Average\ Annual\ Cash\ Flow)$$

Year	Project A	Project B
0	-\$5,000	-\$5,000
1	\$1,000	\$1,544
2	\$2,000	\$1,544
3	\$3,000	\$1,544
4	\$4,000	\$1,544
5		\$1,544
6		\$1,544
7		\$1,544
8		\$1,544
9		\$1,544
10		\$1,544
11		\$1,544
Average	\$1,000	\$999

The convention is that negative values are costs and positive values are revenues.

Here Project A and B have the same payback period (close to five years), but most investors would prefer Project A because the returns are clustered closer to the present.

The payback period is a quick but imprecise measure of investment worth since it ignores future returns after the breakeven is met. It also biases decisions for a quick return.

The net nominal returns are essentially the same.

Module 8: Economic Decision: Time value of money and economic decisions

See

Time Value of Money – Future Value.xlsx

Time Value of Money – Present Value.xlsx

Make sure to work through the exercises to understand how to calculate the payback period in Excel.

Present value methods
 The further in the future a benefit or cost will be received, the less its present value. To the “rational” investor, the present value of \$100 to be received in three years is usually worth less than the value of \$100 received right now.
 The present value formula shows this.

$$PV = \sum_{i=1}^n F_i / (1+r)^i$$

- Example: \$100 received in four annual payments on Jan. 1 of each year is presently worth \$348.60.
 $P = \$100 / (1+.1)^0 + \$100 / (1+.1)^1 + \$100 / (1+.1)^2 + \$100 / (1+.1)^3$
 $P = \$100 + \$90.9 + \$82.6 + \$75.1 = \$348.6$ (much less than \$400)

The Excel formula for present value is **=PV(rate, npr,amt,(fv),type)**, where the rate is the interest rate; npr is the number of payments; amount is the payment; fv is optional and the balance at the end of a period, which is usually 0, such as in paying off a loan; and type is payment timing, with 0 at the beginning of the period (Jan. 1) or end of the period (Dec. 31). For the example above, the Excel formulation is

-348.69	=PV(0.1,4,100,,1)
-316.19	=PV(0.1,4,100,,0)

In Excel, a negative amount is a payment made, as opposed to cash received. For loan payments leave the formula as is, for cash receipts multiply by -1 to calculate the present value.

Module 8: Economic Decision: Time value of money and economic decisions

Note:

Discounted present value of a cash payment of \$10,000		
Year	5%	7%
1	\$9,523.81	\$9,345.79
2	\$9,070.29	\$8,734.39
3	\$8,638.38	\$8,162.98
4	\$8,227.02	\$7,628.95
5	\$7,835.26	\$7,129.86
Total	\$43,294.77	\$41,001.97

Imagine that you will receive \$10,000 a year for the next five years and that $i = 5\%$. To calculate the present value of this stream of income, we simply calculate the present value of each year and add.

$$\begin{aligned}
 & \$10,000/(1.05)^1 + \\
 & \$10,000/(1.05)^2 + \\
 & \$10,000/(1.05)^3 + \\
 & \$10,000/(1.05)^4 + \\
 & \$10,000/(1.05)^5 = \\
 & \$9,523 + \$9,070 + \$8,638 + \$8,227 + \\
 & \$7,835 = \$43,293
 \end{aligned}$$

\$10,000 received each year for five years has a present value of **\$43,295** if the interest rate = 5%. If $i = 7\%$, the PV = **\$41,002**.

For some simple experiments with present value, see

[Present Value Models.xlsx](#)

[Present Value Example.xlsx](#)

Net present value

Calculate the net present value of an investment by using a discount rate and a series of future payments (negative values) and income (positive values). To translate this into benefit/cost analysis, each period has a benefit (revenue) and a cost, and forms the net benefit $(B_i - C_i)$ for period i . Some periods may have no benefits or costs, and the value of $(B_i - C_i)$ can be negative, zero, or positive.

$$NPV = \sum_{i=1}^n (B_i - C_i) / (1 + r)^i$$

The Excel formula for net present value =**NPV(rate,value1,[value2]...)**, where the rate is the interest rate and Value 1, Value 2, etc. are the net benefits for each period. The periods must have the same “space” (i.e., no mixing monthly and annual data), and be ordered in time.

See

[Discounted Cash Flow.xlsx](#)

Video: [Discounted cash flow](#)

Module 8: Economic Decision: Time value of money and economic decisions

Present Value of \$1000 received at different times in the future at various interest rates						
Interest (i)→	2%	4%	6%	8%	10%	12%
Year(n)↓						
0	\$1,000	\$1,000	\$1,000	\$1,000	\$1,000	\$1,000
1	\$980	\$962	\$943	\$925	\$908	\$893
2	\$961	\$925	\$890	\$857	\$826	\$797
3	\$942	\$889	\$840	\$794	\$751	\$712
4	\$924	\$855	\$792	\$735	\$683	\$636
5	\$906	\$822	\$747	\$681	\$621	\$567
6	\$888	\$790	\$705	\$630	\$564	\$507
7	\$871	\$760	\$665	\$583	\$513	\$452
8	\$853	\$731	\$627	\$540	\$467	\$404
9	\$837	\$703	\$592	\$500	\$424	\$361
10	\$820	\$676	\$558	\$463	\$386	\$322
15	\$743	\$555	\$417	\$315	\$239	\$183
20	\$673	\$456	\$312	\$215	\$149	\$104
25	\$610	\$375	\$233	\$146	\$92	\$59
30	\$552	\$308	\$174	\$99	\$57	\$33
Total	\$12,560	\$10,806	\$9,496	\$8,485	\$7,682	\$7,029

Year 0 is the start of the year (January 01), Year 1 is the end of the year (December 31). Pay attention to the problem context.

The formula in any column is $PV = FV / (1+i)^n$

The present value of \$1,000 received every year for 15 years with $i = 2\%$ is \$12,560.

The present value of \$1,000 received at the end of Year 8 with $i = 6\%$ is \$540.

3. Working with the formulas

The formulas $(1+r)^n$ and $1/(1+r)^n$ support a range of specialized roles, such as

- saving to reach a target
- creating a sinking fund
- annuities
- capital recovery (mortgages)

3.1. Saving to reach a target

Often a need exists to set aside funds to reach a target amount. Imagine setting aside a sum annually to reach \$5,000.

Assuming you can invest in some form of

bond with a positive interest, you can set aside less than \$5,000 a year since the first deposit will

The formulas in this section use the same core Excel function, $=PMT(\text{rate}, \text{nper}, \text{pv}, (\text{fv}), (\text{type}))$, where

- rate is the effective interest
- nper is the number of periods (typically in months)
- pv is the present value
- fv is the future value
- type is 0 or 1 depending on whether payments are due at the beginning or end of the months

Depending on the arguments entered, the formula adapts to each of the problem types. See the Annex and the Help option in Excel $F1 \rightarrow =PMT$ to see this in action.

Module 8: Economic Decision: Time value of money and economic decisions

earn interest for five years, the second earns interest for four years, and so on. If A is the annual amount to be set aside, F the target sum, and r = the interest rate, then the formula linking the annual amount, the future amount, and interest is

$$A = F / (1+r)^n$$

- Example: The amount needed to save \$5,000 in six years if the interest rate available is 2.6% is $A=5000/(1.025)^6 = \$4,311.48$. To check, use the formula.

$$F = A(1+r)^n \text{ or } \$4311.48(1.025)^6 = \$4311.48(1.1597) = \$4999.99.$$

- Example: What is the interest implied by an annual payment A=\$1200 reaching \$8,600 in seven years? Rearranging terms results in

$$r = \sqrt[n]{\frac{F}{A}} - 1$$

$$= \left[\frac{F}{A} \right]^{1/n} - 1$$

Note: We can use the Excel formula =POWER(Number, 1/root) to solve this formula. Assume that cell A2 has the value F/A, and that cell A3 has the value 1/n, then =POWER(A2,A3) = A2^(1/A3). For the problem above $r = 1.324$. As an exercise, create a small Excel sheet with cells containing F, A, and n, and then use the =POWER function to find r as you vary F, A, and n.

3.2. Sinking fund

Imagine we needed to replace computers in a business every five years and each cost \$1,000 for a total of \$5,000. One approach is to set aside \$1,000 per year for five years. But since money will earn interest starting in the first year, this approach will overshoot the goal.

A **sinking fund** accumulates this target amount of money but with a lower annual contribution. The first payment at the end of Year 1 will earn interest for four years. Similarly, the second payment at the end of Year 2 will earn interest for three years.

A sinking fund exploits the fact that deposited funds earn interest that results in

- lowering annual payments, so that less needs to be set aside each year, or
- reaching the target in less time.

The sinking fund formula appears as

$$A = F / ((1+r)^n - 1)$$

Module 8: Economic Decision: Time value of money and economic decisions

where A is the annual payment, F the target value, r the interest rate, and n the number of periods available to accumulate the target value.

3.3. Compound amount factor (sinking fund in reverse)

This formula computes the future amount that annual payments of A will grow to in n periods if the interest rate is r.

$$F = A[(1+r)^n - 1 / r]$$

Note the relationship between this formula and the sinking fund formula.

- **Example:** 60 annual payments of \$1,300 will grow to what if the annual interest rate is 3.5%?
 - Use the Excel formula =PMT but specifying the *rate* and *nper* is tricky. If the payments are monthly and you are quoted an annual interest of r, then rate = r/12. If the loan is for four years, then nper = 4*12 = 48.
 - The term pv reflects the present value of the series of payments at the start. Most often, set this to 0, and use “,”.
 - Type = 0 starts the process on Dec. 31 of Year 1, and Type = 1 starts it on Jan. 1 of Year 1. In this example, F = \$255,471.95 (enter rate as .035).
- **Example:** A regular series of \$50 payments each year will eventually accumulate to \$4,917 after 25 years if r =10%.

$$\$4,917 = \$50 ((1.10)^{25} - 1) / .10 = FV(.10, 25, 50, 0)$$

- This formula shows that a future value of \$4,917 will result in 25 years if we invest \$50 per year.
- If we merely discount the future value (\$4,917) into the present (by applying the present value factor), we can then discover how much a regular \$50 payment is worth now.

See

Sinking Fund 1.xlsx

Sinking Fund 2.xlsx

3.4. Annuity

An annuity is any regular payment, such as wages, rent, and quarterly stock dividends. The time between payments is the *interval*, with the most usual form being an *ordinary annuity certain*,

Module 8: Economic Decision: Time value of money and economic decisions

which is a fixed payment every interval with a defined end point. The valuation of the annuity is simply the present value of the regular payments. For example, an annuity of \$500 (annual) received for four years with an interest rate of 5%. If the annuity is “banked,” the value S is

$$\begin{aligned} S &= 500(1.05)^3 + 500(1.05)^2 + 500(1.05)^1 + 500 \\ &= 500 + 500(1.05)^1 + 500(1.05)^2 + 500(1.05)^3 \\ &= 500(1 + 1.05 + 1.025 + 1.1576) \\ &= 500(4.3101) = \$2155.10 \end{aligned}$$

The present value of the payments is simply the discounted stream of payments. Annuities have two formulas: S, the value of the annuity (the purchase value), and P, the present value of an annuity. In this example, an annuity of \$2,155 (at 5%) will produce annual payments of \$500 for four years. A simple savings account that pays 5% would yield \$107 annually, but of course you have the \$500 at the end.

As mentioned, retirees will often purchase a large-value annuity (more than \$50,000), which will run until death. Depending on the age of purchase and the age at death, this can be a favourable investment.

The two formulas for an annuity are for S (the purchase value of the annuity) and P (the present value of the annuity).

$$S = R \frac{(1+i)^n - 1}{i}$$

where R is the periodic payment

In Excel S is given by

=FV(Rate, Nper, Pmt, PV, Type)	
Rate	Interest per period
Nper	Total number of payment periods
Pmt	Payment
PV	Present value of the annuity (Optional)
Type	= 0 for payment at the end of period, and 1 for payment at start; if omitted, assumes payment at end

Example: A regular series of \$50 payments each year will eventually accumulate to \$4,917 after 25 years if $i=10\%$.

$$\$4,917 = \$50 \left(\frac{(1.10)^{25} - 1}{.10} \right) = FV(.10, 25, 50, 0)$$

Module 8: Economic Decision: Time value of money and economic decisions

- This formula shows that a future value of \$4,917 will be obtained in 25 years if we invest \$50 per year.
- If we merely discount the future value (\$4,917) into the present (by applying the present value factor), we can then discover how much a regular \$50 payment is worth now.

The present value of an annuity is given by the formula

$$P = S \frac{(1+i)^n - 1}{i(1+i)^n}$$

The Excel formula for P is

=PPMT(Rate, Per, Nper, PV, FV(optional))	
Rate	Interest per period (effective interest)
Per	Must be between 1 and Nper
Nper	Total number of payment periods (months)
PV	Present value of the annuity
FV	Cash balance after the last payment (optional)

Example: If I buy an annuity for \$12,000, and I want 10 years of payments, how much can I expect each year if the interest is 5%?

$$= \$12,000 \left(\frac{(1.05)^{10} - 1}{.05(1.05)^{10}} \right)$$

$$= \text{PPMT}(.05, 1, 10, 12000, , 1) = \$1,480.05$$

Example: If I buy a car for \$12,000, and I want to pay over five years of payments, how much can I expect to pay each year if the interest is 5%?

$$= \$12,000 \left(\frac{(1.05)^5 - 1}{.05(1.05)^5} \right)$$

$$= \text{PMT}(.05, 5, 12,000, , 1) = \$2,639.71$$

Example: If I buy a car for \$12,000, and I want to pay over five years of payments, how much can I expect to pay each month if the annual interest is 5%? (Note that there are 60 payments with a monthly interest of $.05/12 = .42\%$).

$$= \$12,000 \left(\frac{(1.0042)^{60} - 1}{.0042(1.0042)^{60}} \right)$$

$$= \text{PMT}(.0042, 60, 12000, , 1) = \$225.73$$

Notice how we need to change the annual interest to a monthly interest.

Module 8: Economic Decision: Time value of money and economic decisions**3.5. Capital recovery factor (Mortgage)**

Closely related the annuity is the capital recovery factor, used to calculate the monthly payments to settle a loan. The capital recovery factor enables one to calculate the yearly payments required to retire a debt of P in n years. In each year, a portion of the repayment is a retirement of the principal, while a portion is a payment to the interest on the remaining loan. The annual payment is

$$A = P \frac{1(1+i)^n}{(1+i)^n - 1}$$

where P is the amount of the loan and n and i are as previously defined.

The Excel formula is

=PMT(Rate, Per, Nper, PV, FV(optional))	
Rate	Interest per period
Per	Must be between 1 and Nper
Nper	Total number of payment periods
PV	Present value of the annuity
FV	Cash balance after the last payment (optional)

Example. A loan of \$10,000 to be repaid over 20 years if the interest rate is 10 % requires annual payments of \$1,174.60.

$$=\$1,174.60 = \$10,000 (.10(1.10)^{20}) / (1.10)^{20} - 1$$

Video: [Mortgages](#)

See

[Mortgage Example.xlsx](#)

for some additional “tricks” to incorporate row referencing in formulas.

4. Economic decisions

Economics attempts to create an internally consistent system for choosing among alternative ways to allocate scarce resources to meet a specific goal. Some decisions occur within a narrow technical setting with cost minimization, revenue maximization, market share enhancement, or profit maximization as the goals. In other cases, such as healthcare, the extension of life, reduction of disease, or the maximization of the quality of life become the goals. Students choose to allocate their most scarce resource – time – among courses designed to meet their goals, secure a career, increase knowledge, extend their capacity to reason.

Module 8: Economic Decision: Time value of money and economic decisions

Case 1: A bus company is considering a new piece of equipment to reduce costs. Its present annual cost is \$20,000 and there are several options open.

- **Plan A** is to do nothing (this is *always* an option).
- **Plan B** is to buy a machine to reduce costs by \$5,000 per year. It has an initial cost of \$30,000. It will also cost \$20,000 to install the machine, and in the first year there will be no reduction in operating costs.
- **Plan C** is to purchase another type of machine costing \$40,000, requiring no time to install (or money), which immediately yields a saving of \$2,000 per year.

Case 2: Leslie is considering three options for university. Assume that room and board and other essentials (coffee, entertainment, etc.) will cost \$15,000 annually.

- **Plan A** is to not go to university and work for Skip the Dishes.
- **Plan B** is to complete a general BA over four years at a tuition cost of \$7,500 annually. The expected post-university job will pay \$32,000 annually, increasing by 2% per year with a working life of 30 years.
- **Plan C** is to complete a science degree over four years with annual tuition at \$9,000, enter med school (three years at \$35,000 annually), complete a speciality in family medicine of one year (\$35,000 annually), and then complete a residency (paying \$45,000 annually), and enter practice at \$180,000 annually, increasing by 4% per year. It costs \$60,000 to buy into the practice. Assume a working life of 25 years.

4.1. The cost-benefit model

Cost-benefit analysis (CBA) – or equivalently, benefit-cost analysis (BCA) – represents a standard method for evaluating investment projects with alternative revenue and cost profiles. The essential feature is that all costs and all benefits must find expression in financial terms. Many benefits and costs are clear and easily translated into cash terms, such as the two cases. Others are indirect and inherently hard to define and measure, such as the bus fare incurred by a student who sells their car to pay tuition. Other costs may be intangible, such as the constrained social life of someone trying to get a date when their mode of transport is a bicycle.

Module 8: Economic Decision: Time value of money and economic decisions

Many economic and business models may be viewed as cash flows (positive and negative over time). Cash appears on the vertical (Y axis).

Costs appear as negative values on the Y axis and revenues as positive values. The X axis represents time.

It can be tricky to decide how to count time. One approach is to define the start of a project at 00:01 on January 1 (Year 0) and then define Year 1 as 23:59 on the next December 31. Just make your assumptions clear.

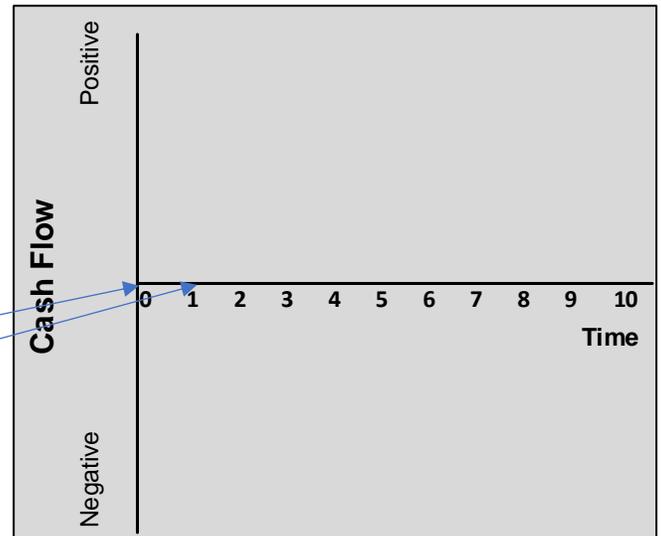


Figure 2: Economic decision horizon

To illustrate basic principles, we can use simple measures of time, such as year, pretending the initial cost occurs at the start of a project (Year 0), and using bar charts to represent revenues and costs.

It is possible to visualize a project as cash flows over time. Income appears as a positive (above 0) and expenses as negative (below the line).

Project A shows an initial investment in Year 0 that results in a stream of positive, but declining revenues in Years 1 to 10. Note that the graphs obscure the time detail. Year “0” has three interpretations:

- All spending occurs at one second after midnight on January 1.
- A little spending occurs each day of the year.
- All spending occurs on December 31 of Year 0.

It is important to be consistent in how you designate time. One could start with Year 1 and not Year 0. Just be clear and consistent about the timing of cash flows.

Module 8: Economic Decision: Time value of money and economic decisions

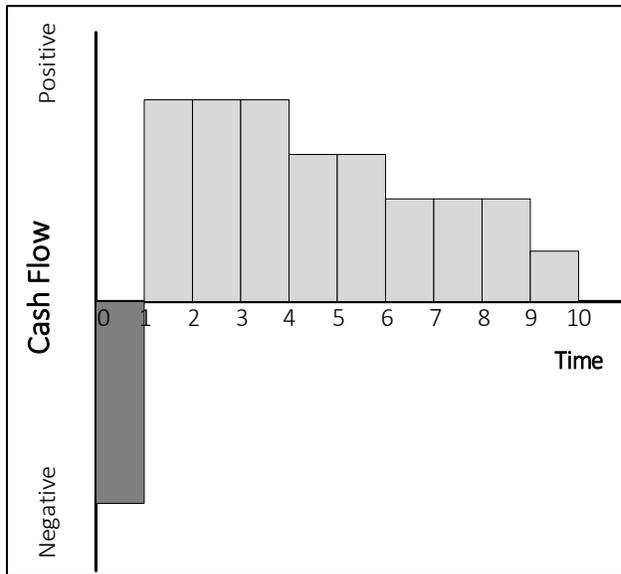


Figure 3: Cost/revenue profile Project A

In contrast, **Project B** shows a situation where management performs an equipment upgrade in Year 5, resulting in an increase in revenues, which recover, then decline again. Note the spending in Year 5 to show that investment.

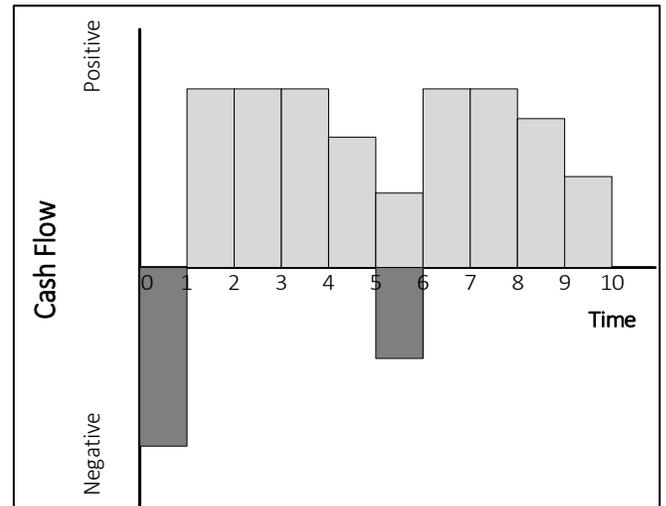


Figure 4: Cost/revenue profile Project B

In Figure 5, management makes an initial investment, but also has operating expenditures each year. These increase as equipment ages.

The investment in Year 5 increases revenues in Year 6 but also reduces operating expenditures in Years 6–8, which then rise in Years 9 and 10.

The dark line shows the path of net revenues, the formula of which is

$$\begin{aligned} \text{Net Income} &= (B_0 - C_0) + (B_1 - C_1) + \dots + (R_{10} - C_{10}) \\ &= \sum_{i=1}^n (R_i - C_i) \end{aligned}$$

It becomes challenging to visualize complex expenditure patterns, and Excel is a preferred approach.

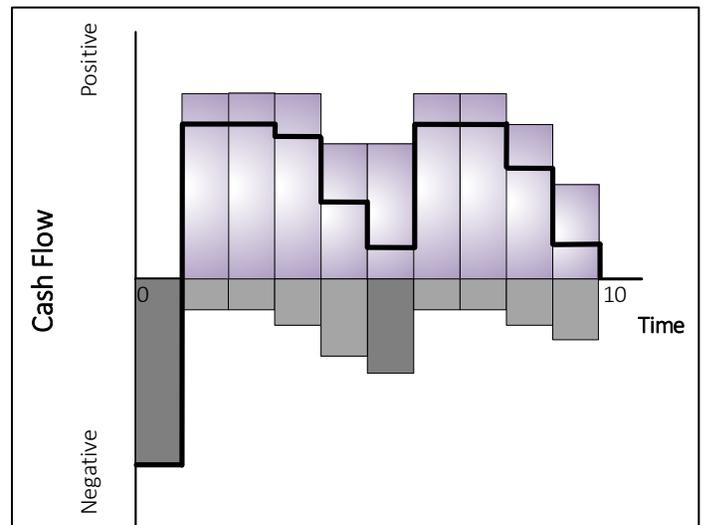


Figure 5: Figure 5 Net revenue (Benefits)

See

[NPV, IRR and BC Experiments.xlsx](#)

for some simulation exercises on net present value.

Module 8: Economic Decision: Time value of money and economic decisions

The CBA basic model compares the discounted benefits (measured as monetary values) to the discounted costs (also measured in monetary values). The cost-benefit model only uses money equivalents for benefits and costs, which is usually quite straightforward in a business context, but can be challenging when social costs and benefits enter the calculation, as is common in public sector decisions. Even the decision faced by Leslie in choosing whether to attend university and then which course to take may all have forms of intangible dimensions. For example, parental pressure to become a doctor might be intense, and the “psychological” costs of choosing no education and a minimum wage job could be high.

A complete CBA needs to find a way to translate these psychological costs into money, which can be challenging. This is an important complication/qualification of CBA, the exploration of which lies beyond the scope of this text.

Discounting revenues/costs requires three steps:

- setting an interest rate
- deciding a project horizon (time)
- projection/forecasting revenues/costs over the project horizon

The key idea is that future revenues and costs experience discounting according to the present value formulation. Costs/revenues that lie further in the future have a lower present value than the costs/revenues that lie closer to the start of a project.

Video: [Basic BC Ratio](#)

Note that we use “revenues” in a business context to denote financial flows, and “benefits” in a government context, reflecting the indirect/intangible nature of many public projects. Provided we create monetary equivalents of indirect/intangible costs, we can combine these costs/benefits with financial cost and revenues in the same model.

The sum of discounted revenues = $\sum_{i=1}^n \frac{R_i}{(1+r)^i}$ and the sum of discounted costs = $\sum_{i=1}^n \frac{C_i}{(1+r)^i}$.

The *internal rate of return* is interest rate “r” that satisfies the following equation

Module 8: Economic Decision: Time value of money and economic decisions

$$\sum_{i=1}^n \frac{R_i}{(1+r)^i} - \sum_{i=1}^n \frac{C_i}{(1+r)^i} = 0 \text{ or equivalently}$$

$$\sum_{i=1}^n \frac{R_i - C_i}{(1+r)^i} = 0$$

Any stream of benefits (positive) and costs (negative) has an internal rate of return (IRR). Generally, if benefits exceed costs, the internal rate of return will be positive, **but not always**.

Example: If costs cluster at the start, and benefits are very delayed, then the internal rate of return may be negative.

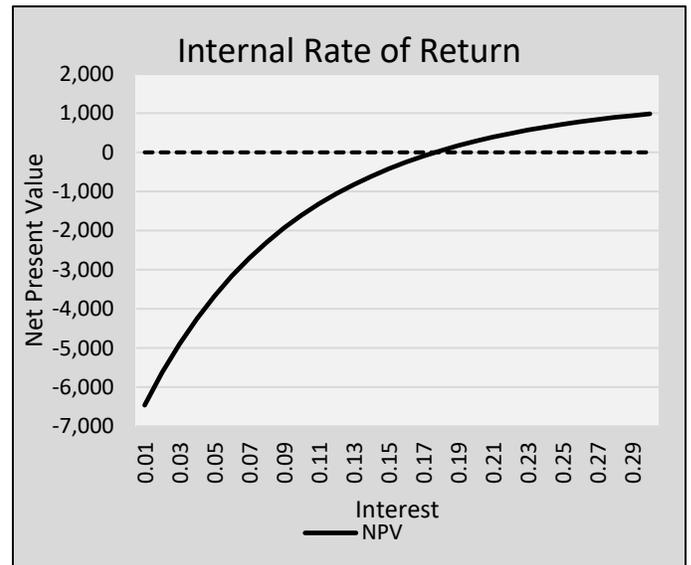


Figure 6: Internal rate of return

See

[BasicBC Ratio.xlsx](#)

[Internal Rate of Return – Graphical.xlsx](#)

for some simulations.

4.2. Costs

Costs of both public and private sector projects fall into three categories – direct, indirect, and intangible.

Direct costs include variable and fixed costs:

- **Variable costs** (labour, supplies, energy...) are 0 when production ceases and increases in step (usually non-linearly) with output.
- **Fixed costs:** Cost of equipment, new construction, rent on the building, etc. that do not vary with output and are either paid in a lump sum at the start or spread out over time through leasing or loan repayments.

Module 8: Economic Decision: Time value of money and economic decisions

Indirect costs include *opportunity cost, depreciation, externalities, and consequential costs*.

- **Opportunity cost** is the lost revenue from an alternative activity or revenue. Since we assume that rational investors (including students) always commit resources to the activity with the highest return, we usually do not include this as a cost.
- **Depreciation** reflects the loss in value of equipment through wear and tear and obsolescence. In earlier times, firms may have created a sinking fund to replace the capital at a future date. The payment to the fund is a cost. Now, many tax regimes allow a “paper” deduction against income as a support measure for the erosion of capital in a business. This reduces net income and tax liability, but these charges may not be *included in the calculation of the internal rate of return*.
- **Externalities** are another form of indirect impact. They may be positive, such as firms purposely locating within a high-tech cluster where employees transfer among firms or the positioning of complementary activities, such as beekeeping and agriculture. One might add this to revenues if it were possible to place a financial value on such benefits. Often externalities are negative, such as a firm that pollutes a waterway, increasing costs for downstream users. Potential downstream users may need to increase their costs when calculating benefit-cost values, or government may impose penalties on the firm that it must include in its calculations.
- **Consequential costs:** When Leslie sells the car to pay tuition, this triggers a cost of bus fare and/or a bicycle. These costs are a consequence of deciding on more education, and whether they are justified depends on the full analysis.

Intangible costs are difficult to compute because they are intangible. An example is the pain and suffering associated with injuries. A CBA of traffic safety investments could attempt to predict and value the injury reduction.

Module 8: Economic Decision: Time value of money and economic decisions

5. Creating economic and business models in Excel

As a prelude to the optimization work in Module 10, study the spreadsheet which are quite simple but offer insight into the core principles of creating models in Excel.

Really Basic Economic Models.xlsx

Certain strategies are useful in creating economic models in Excel.

1. Use a dashboard to support systematic changes to the model parameters. Using a standard color to designate dashboards is always helpful.
2. Make sure that the cells referencing model parameters use fixed cell references.

Model 1 - Simple revenue

This just creates a revenue function based on price (Cell A3) and sales or quantity sold (Cell B3). The result appears in Cell C3. Changing the parameters (Cells B15 and D16) updates the model.

Model 2 - Profit as a function of sales, price, and unit cost

This model adds a simple unit cost. Unit cost is less than unit price. Increasing sales will always produce more profit.

Model 3 – Profit as a function of sales, price, and unit costs that increase with sales

Here the unit costs rise with sales. (Mouse over cell D13 to reveal the formula.) At some level of sales, profits will become 0 and then negative. What is the breakeven level of sales (where profit becomes 0)?

	A	B	C	D	E	F
1	Model 1 - Simple revenue					
2	Widgets Sold	Price per widget	Total Revenue			
3	1	10	10			
4						
5	Model 2 - Profit as a function of sales, price and unit cost					
6				Profit (revenue - costs)		
7	Widgets sold	Price per widget	Cost per widget			
8	1	\$10	\$8	\$2		
9						
10	Model 3 - Profit as a function of sales, price and unit cost (increasing)					
11				Profit (revenue - costs)		
12	Widgets sold	Price per widget	Cost per widget			
13	1	10	8.1	1.90		
14						
15	Widgets Sold	1	Price per widget	10	Cost per widget	8
16						
17						
18						
19						

Formulas shown in blue boxes:

- $=A3*B3$ (Cell C3)
- $=A8*B8 - A8*C8$ (Cell D8)
- $=8 + .1*A13$ (Cell D13)
- $=\$B\15 (Cell D15)

Changing cells B16, D15, and F15 change the model parameters to support exploration of assumptions.

6. Summary

This Module introduces the time value of money and how it supports the notion of economic decisions involving cash flows over time. The mark of an economist is the instinctive use of core

Module 8: Economic Decision: Time value of money and economic decisions

ideas such as the distinction between real and nominal values and how the present value of a future cash return reflects assumptions about interest rates and the future financial flows.

Annex: Key Excel functions

Logical functions		
Function	Explanation	Example
ISBLANK()	Tests whether the cell (or range) is empty or not. Note that “blank” is not “0.” This is an extremely easy mistake to make.	=ISBLANK(A2) returns 1 if true, 0 if false.
ISNUMBER()	Tests whether the contents have a number and are not blank.	=ISNUMBER(A2: A4) returns 1 if cells A2 to A4 all have numbers, otherwise it will return 0. (Note that numbers and text are different. This function will not include cells with text.) Make sure that you format the cells as numbers since an entry of “5” will be read as text if the cell is formatted as text – right-click on the cell(s) and make sure.
ISTEXT()	Tests whether the contents have text and are not blank.	=ISTEXT(A2: A4) returns 1 if cells A2 to A4 all have text, otherwise it will return 0. (Note that text and numbers are different.)
ISODD()	Tests whether the contents of a range have any odd values (non-divisible by 2).	=ISODD(A2: A4) returns 1 if cells A2 to A4 all have odd numbers, otherwise it will return 0.
ISEVEN()	Tests whether the contents in the range are even (divisible by 2).	=ISEVEN(A2: A4) returns 1 if cells A2 to A4 all have even numbers, otherwise it will return 0.
ISERROR()	Tests whether a calculation produces an erroneous or illegal value.	=ISERROR(A2/A4, 0) returns the value of 0 if A2/A4 produces an error, usually #DIV/0! See the mortgage calculator example.

Module 8: Economic Decision: Time value of money and economic decisions

Logical functions (cont'd)		
Function	Example	Explanation
IF	IF(object is true, do something, otherwise do something else)	=IF(C1+B2>10,E1=1,E1=2) (If the contents of C1 plus B2 are greater than 10, make E1 = 1; if false, make E1 = 2.)
TRUE	=TRUE(A2<A3)	=TRUE (A2<A3) In the cell where this appears, if A2 is less than A3, place 1 in the cell, otherwise (if A2 not <A3), place 0 in the cell. (Note the placement cell cannot be A2 or A3.)
FALSE	=FALSE(A2<A3)	=FALSE (A2<A3) In the cell where this appears, if A2 is less than A3, place 0, otherwise (if A2 not <A3), place 1. (Note the placement cell cannot be A2 or A3.)
OR	=OR(A2<A3,E1>E2)	=OR(A2<A3, E1>E2) If either is true, it returns the value 1, otherwise it returns 0.
AND	=AND(A2<A3,E1>E2)	=AND(A2<A3, E1>E2) If either both or one is false, it returns the value 0, otherwise (both must be true) it returns 0.

Financial functions		
Function	Arguments	Explanation
EFFECT()	=EFFECT(Nominal_rate,Npery)	=EFFECT(4%,12) will return the effective interest rate for an annual rate of 4% compounded monthly.
NOMINAL()	=NOMINAL(Effect_rate,Npery)	=NOMINAL(.3%,12) will return the annual rate of a .3% effective rate compounded monthly. Note: effective rate=(1+(nominal_rate/Npery))*Npery-1.
FV()	=FV(Rate,Nper,Pmt,PV,Type)	=FV(4%/12,20,-100,,) is the future value of \$100 deposited monthly for 20 months. PV and type are optional. Note that we divide an annual interest of 4% by 12 to get monthly interest. Other formulas below do this as well.
PV()	=PV(Rate,Nper,Pmt,FV,Type)	=PV(4%/12,20*12,500,,) returns the present value of monthly deposits of \$500.

Module 8: Economic Decision: Time value of money and economic decisions

Financial functions		
Function	Arguments	Explanation
PMT()	=PMT(Rate,Nper,PV,FV,Type)	=PMT(4%/12,240,\$100,000,,) returns the monthly payment (principal plus interest) needed to pay off a loan of \$100,000 in 20 years.
PPMT()	=PPMT(Rate,Per,Nper,PV,FV,Type)	=PPMT(4%/12,57,240,100,000,,) returns the principal paid on the 57 th payment out of 240 (20 years * 12). Note 1<Per<=Nper.
IPMT()	=IPMT(Rate,Per,Nper,PV,FV,Type)	=IPMT(4%/12,57,240,100,000,,) returns the annual interest paid on the 57 th payment out of 240 (20 years * 12). Note 1<Per<=Nper.
RATE()	=RATE(Nper,Pmt,PV,FV,Type)	=RATE(240,-300,50000,,) returns the annual interest on a loan of \$50,000 that generates \$300 per month over 20 years.

Miscellaneous functions		
Function	Arguments	Explanation
ROW()	ROW(reference)	ROW(C8) = 8 (This may seem a useless function but see the mortgage example for how it may be used.)
COL()	COL(reference)	Performs as the ROW() function.